CH 5 : OSCILLATIONS

Q. 1 : Define the terms with two examples.
1) Periodic motion 2) Oscillatory motion.

Ans : 1) Periodic Motion : A motion that repeats itself at definite intervals of time is said to be periodic motion.
eg. Motion of hands of a clock, Motion of moon around the earth.

2) Oscillatory motion (Vibratory motion) : A periodic motion in which a body moves back & forth over the same path, straight or curved, between alternate extremes is said to be an oscillatory motion.
eg. Motion of pendulum of clock, Oscillations of spring, swinging of swing.

Note : Every oscillatory motion is periodic but every periodic motion need not be oscillatory circular motion is periodic but it is not oscillatory.

Q. 2 : Define linear simple harmonic motion. Give two examples.

Ans : It is defined as the linear periodic motion of a body, in which the force (or acceleration) is always directed towards the mean position & it's magnitude is proportional to the displacement from the mean position.
eg. Vibrations of prongs of tuning fork, Oscillations of needle of sewing machine.

Q. 3 : Define following terms : 1) Period 2) frequency 3)Amplitude 4) path length of simple harmonic motion (SHM).

Ans : 1) Period (Periodic time) : The time taken by a particle performing simple harmonic motion to complete one oscillation is called the period of SHM.

2) Frequency : The number of oscillations performed per unit time by particle executing SHM is called the frequency of SHM.

3) Amplitude : The magnitude of the maximum displacement of a particle performing SHM from it's mean position is called the amplitude of SHM.

4) Path length : The length of the path over which a particle performs SHM is twice the amplitude of the motion is called path length or range of SHM.

Q. 4 : Form the differential equation of linear S.H.M.

<u>OR</u>

Obtain the differential equation of linear simple harmonic motion. (2M – Mar'24)

Ans : Suppose a particle of mass 'm' is performing S.H.M. 'f' is the restoring force on particle when its displacement is x.
By definition of S.H.M.

$$f \propto x$$

$$\therefore f = -k\,x \quad \ldots\ldots\ldots\ldots(1)$$

where, k = force constant
-ve sign indicates force is opposite to displacement in direction.

According to Newton's second law of motion,

$$\therefore f = ma$$

$$\therefore ma = -k\,x \quad \ldots\ldots\ldots\ldots(2) \qquad [\text{From } (1)]$$

The velocity of particle is, $v = \frac{dx}{dt}$ &
It's acceleration is –

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$\therefore \text{ Equation (2)} \Rightarrow m\left(\frac{d^2x}{dt^2}\right) = -k\,x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\therefore \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Substituting $\frac{k}{m} = \omega^2$, where ω is the angular frequency.

$$\therefore \boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

This is the differential equation of linear SHM.

Q. 5 : State the differential equation of S.H.M. Hence obtain the expressions for acceleration, velocity & displacement of a particle performing S.H.M. [4M – Mar'18]

Ans : The differential equation of S.H.M. is -

$\therefore \frac{d^2x}{dt^2} + \omega^2 x = 0$ ....(1)

Expression for Acceleration :

$\therefore \frac{d^2x}{dt^2} = -\omega^2 x$

But a = $\frac{d^2x}{dt^2}$ is the acceleration of the particle performing S.H.M.

$\therefore$ Acceleration, a = $-\omega^2 x$

Expression for Velocity :

Equation (1) ⇒

$\frac{d^2x}{dt^2} = -\omega^2 x$

$\therefore \frac{d}{dt}\left(\frac{dx}{dt}\right) = -\omega^2 x$

$\therefore \frac{dv}{dt} = -\omega^2 x$

$\therefore \frac{dv}{dx} \cdot \frac{dx}{dt} = -\omega^2 x$

$\therefore \frac{dv}{dx} \cdot v = -\omega^2 x$

$\therefore v\,dv = -\omega^2 x.dx$

Integrating

$\int v\,dv = -\omega^2 \int x dx$

$\therefore \frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + c$ ....(2)

where, c is constant of integration.

At the extreme position, displacement is maximum & velocity is zero.

$\therefore$ At $x = \pm A$, v = 0

$\therefore$ Equation (2) ⇒

$\therefore 0 = -\frac{\omega^2 A^2}{2} + c$

$\therefore c = +\frac{\omega^2 A^2}{2}$

Put this in Equation (2) ⇒

$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$

$= -\omega^2 (A^2 - x^2)$

$\therefore$ $v = \pm \omega \sqrt{A^2 - x^2}$ ....(3)

Expression for Displacement :

Substitute v = $\frac{dx}{dt}$ in equation (3) ⇒

$\therefore \frac{dx}{dt} = \omega \sqrt{A^2 - x^2}$ .....(Considering only magnitude)

$\therefore \frac{dx}{\sqrt{A^2-x^2}} = \omega.dt$

Integrating

$\int \frac{dx}{\sqrt{A^2-x^2}} = \omega \int dt$

$\therefore \sin^{-1}\left(\frac{x}{a}\right) = \omega t + \phi$

where, ϕ = constant of integration.

$\therefore \frac{x}{A} = \sin(\omega t + \phi)$

$\therefore$ **$x = A \sin(\omega t + \phi)$**

Q. 6 : Using differential equation of linear S.H.M. obtain expression for displacement.

Ans. : Write Q. No. (5) – Only **Expression for Displacement** part.

Q. 7 : Obtain the extreme values (maximum & minimum) of displacement, velocity & acceleration of a particle performing SHM.

Ans : 1) Displacement : The general expression for displacement (x) is –

$x = A \sin(\omega t + \phi)$ ....(1)

At mean position, $(\omega t + \phi) = 0$ or π

$\therefore$ Equation (1) ⇒

$x = A \sin(0)$ OR $A \sin(\pi)$

$\therefore$ $x_{min} = 0$

Thus, at mean position, the displacement of particle is minimum (ie. zero).

At extreme position, $(\omega t + \phi) = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$\therefore$ Equation (1) ⇒

$x = A \sin\left(\frac{\pi}{2}\right)$ or $A \sin\left(\frac{3\pi}{2}\right)$

$\therefore$ $x_{max} = \pm A$

Thus, at extreme position, the displacement of particle is maximum.

2) Velocity : The general expression for velocity (v) is –

$$v = \pm\, \omega \sqrt{A^2 - x^2} \qquad \text{........(2)}$$

At mean position, $x = 0$

$\therefore$ Equation (2) $\Rightarrow$

$$v = \pm\, \omega \sqrt{A^2 - 0}$$

$$\therefore \quad \boxed{v_{\max} = \pm A\, \omega}$$

Thus, at mean position, velocity is maximum.

At extreme position, $x = \pm A$

$\therefore$ Equation (2) $\Rightarrow$

$$v = \pm\, \omega \sqrt{A^2 - A^2}$$

$$\therefore \quad \boxed{v_{\min} = 0}$$

Thus, at extreme position, the velocity is minimum.

3) Acceleration : The general expression for magnitude of acceleration (a) is –

$$a = \omega^2 x \qquad \text{........(3)}$$

At mean position, $x = 0$

$\therefore$ Equation (3) $\Rightarrow$

$$a = 0$$

$$\therefore \quad \boxed{a_{\min} = 0}$$

Thus, at mean position, acceleration is minimum.

At extreme position, $x = \pm A$

$\therefore$ Equation (3) $\Rightarrow$

$$v = \pm\, \omega^2 A$$

$$\therefore \quad \boxed{a_{\max} = \pm\, \omega^2 A}$$

Thus, at extreme position, acceleration is maximum.

Q. 8 : Derive expression for period of S.H.M. in terms of 1) angular frequency 2) force constant 3) Acceleration. Also find expression for frequency.

Ans : The general expression for displacement (x) of a particle performing S.H.M. is -

$$x = A \sin(\omega t + \phi)$$

1) Let T be the period of S.H.M. After time t = $\left(t + \frac{2\pi}{\omega}\right)$ the displacement will be –

$$x = A \sin\left[\omega \left(t + \frac{2\Pi}{\omega}\right) + \phi\right]$$

$$= A \sin(\omega t + 2\pi + \phi)$$

$$= A \sin(\omega t + \phi)$$

Thus after time $\frac{2\pi}{\omega}$, the particle is at the same position ie. it has completed one oscillation. Thus it is the minimum time after which it repeats.

$\therefore$ Period is - $\boxed{T = \frac{2\pi}{\omega}}$

2) If m is mass of the particle & k is the force constant, $\omega = \sqrt{\frac{k}{m}}$

$$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}}$$

$$\therefore \quad \boxed{T = 2\pi \sqrt{\frac{m}{k}}}$$

3) The acceleration of a particle performing S. H.M. has a magnitude -

$$a = \omega^2 x$$

$$\omega = \sqrt{a/x}$$

$$= \sqrt{Accel^n \ per\ unit\ displacement}$$

Now, $T = \frac{2\pi}{\omega}$

$$\therefore \quad \boxed{T = \frac{2\pi}{\sqrt{Accel^n \ per\ unit\ displacement}}}$$

The frequency of S H M is –

$$\boxed{n = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Q. 9 : Show that linear S.H.M. is the projection of U.C.M. on it's diameter.

Ans : Consider a particle P moving along circumference of circle of radius A with constant angular speed ω in anticlockwise direction.

At any instant t=0, particle P has it's projection at point M as shown in fig(a). P is called reference particle & the circle on which it moves is called reference circle.

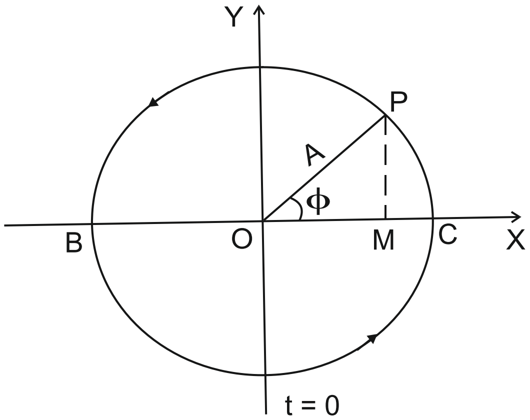


Fig (a)

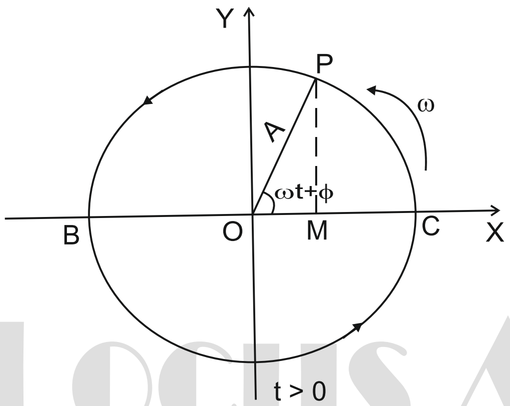


Fig (b)

As P revolves, it's projection moves back & forth about centre O along diameter BC. The x-component of displacement, velocity & acceleration of P is always same as the displacement, velocity & acceleration of M.

Suppose P starts from initial position with phase ϕ. In time t, the angle between OP & x-axis is $(\omega t + \phi)$ as shown in Fig (b).

Here, $\cos(\omega t + \phi) = \frac{x}{A}$, where x is displacement.

$\therefore x = A\cos(\omega t + \phi)$ ...... (1)

This equation represents displacement. Velocity is -

$v = \frac{dx}{dt} = -A\,\omega \sin(\omega t + \phi)$ ...... (2)

This equation represents velocity of projection of P at time t.

Acceleration is –

$a = \frac{dv}{dt} = -A\,\omega^2 \cos(\omega t + \phi)$

$= -\omega^2 x$ ...... (3)

This equation represents acceleration.

From equation (3) =>

$$a \propto -x$$

Thus acceleration is proportional to it's displacement & in opposite direction.

Thus projection of P performs simple harmonic motion. But M is projection of P, performing UCM. Hence, S.H.M. is projection of UCM along a diameter of circle.

Q. 10 : Explain the terms phase & epoch of an S.H.M.

Ans : 1) Phase : The physical quantity which describes the state of oscillation is phase of S.H.M. ie. it gives the magnitude & direction of displacement of particle.

$x = a\sin(\omega t + \phi)$

Here, $(\omega t + \phi)$ is called phase angle or phase of S.H.M.

S.I. unit ⇒ rad

2) Epoch : The physical quantity which describes the state of oscillation of particle performing S.H.M. at the start of motion is called epoch of S.H.M.

In the term phase $(\omega t + \phi)$, ϕ is phase at start of S.H.M. ie. at t = 0. ∴ This angle ϕ is called starting phase or initial phase or epoch of S.H.M.

S.I. unit ⇒ rad

Q. 11 : Particle performing S.H.M. starts from mean position. Plot a graph of displacement, velocity & acceleration against time. OR Explain graphical representation of displacement, velocity & acceleration when particle starts its S.H.M. from mean position towards positive.

Ans : At Mean position, $\phi = 0$

Displacement, $x = A\sin\omega t$

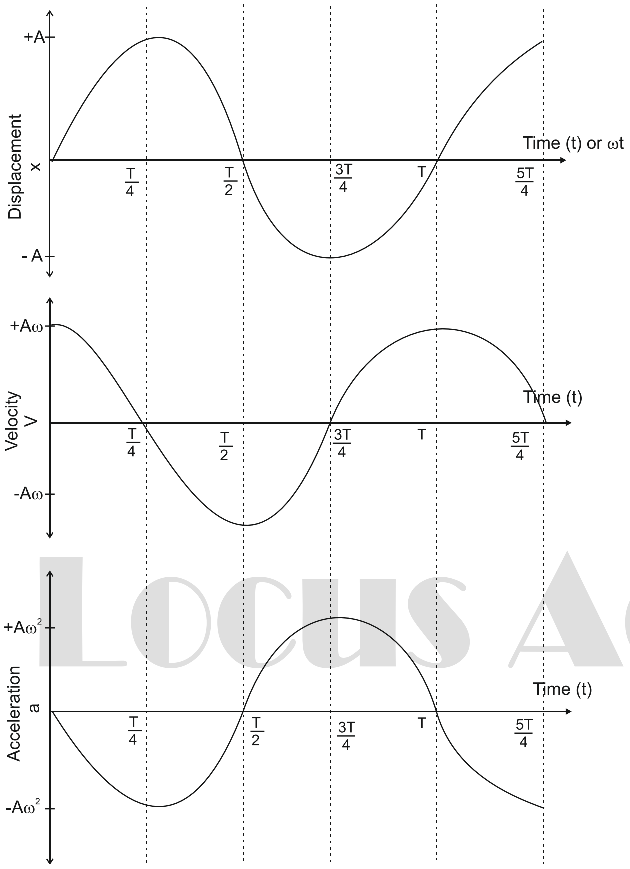
Velocity, $v = \frac{dx}{dt} = A\omega\cos\omega t$

Acceleration, $a = \frac{dv}{dt} = -A\omega^2\sin\omega t$

The values of x, v & a are tabulated as below –

	0	T/4	T/2	3T/4	T	5T/4
ωt	0	$\pi/2$	π	$3\pi/2$	2π	$5\pi/2$
x	0	A	0	-A	0	A
v	Aω	0	-Aω	0	Aω	0
a	0	$-A\omega^2$	0	$A\omega^2$	0	$-A\omega^2$

The graphs of displacement, velocity & Acceleration can be plotted as follows -



Q. 12 : Particle performing S.H.M. starts from positive extreme position. Plot a graph of displacement velocity & acceleration against time. OR

Show variation of displacement, velocity & acceleration with phase for a particle performing linear S.H.M. graphically, when it starts from extreme position. [3M – Oct'14]

Ans : At extreme positive position, $\phi = \pi/2$

Displacement x = A sin (ωt $+\pi/2$) = A cos ωt

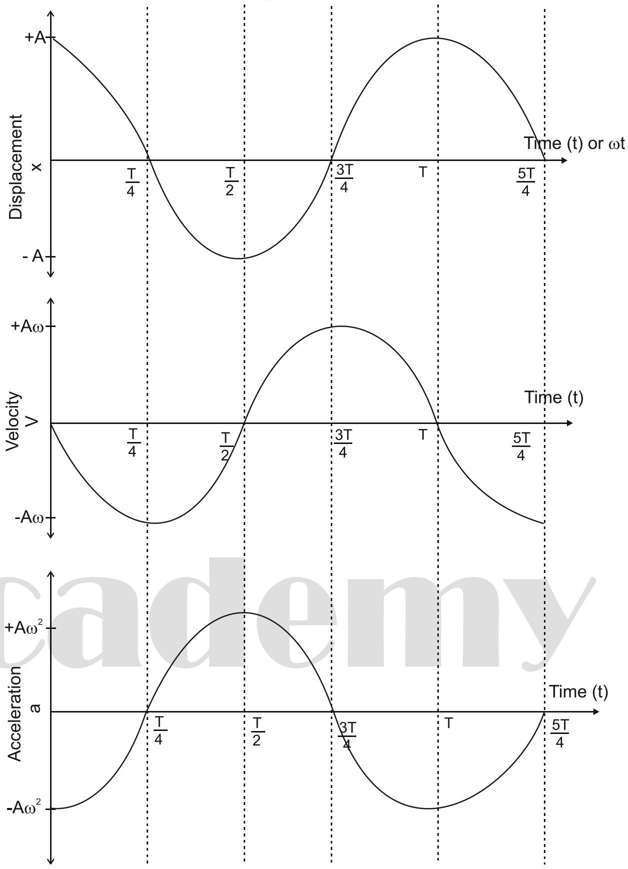
Velocity, v = $\frac{dx}{dt}$ = - Aω sin ωt

Acceleration, a = $\frac{dv}{dt}$ = - $A\omega^2$ cos ωt

The values of x, v & a are tabulated as below –

T	0	T/4	T/2	3T/4	T	5T/4
ωt	0	$\pi/2$	π	$3\pi/2$	2π	$5\pi/2$
x	A	0	-A	0	A	0
v	0	-Aω	0	Aω	0	-Aω
a	$-A\omega^2$	0	$A\omega^2$	0	$-A\omega^2$	0

The graph of displacement velocity & acceleration can be plotted as follows –



Q. 13 : State expressions for displacement, velocity & acceleration of particle performing linear SHM starting from mean position (OR extreme position), Draw your conclusions from the graph.

Ans : From mean position ⇒ Q. No. (11)

From extreme position ⇒ Q. No. (12)

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Conclusions :

1) The displacement, velocity & acceleration of particle are periodic functions. From mean position, x-t and a-t graph are sine curves & v-t graph is cosine curve (from extreme position, v-t graph is sine curve & x-t and a-t graph are cosine curve).

2) There is a phase difference of $\pi/2$ rad between x & v and v & a.
3) There is a phase difference of π rad between x & a.

Q. 14 : Discuss analytically, the composition of two S.H.M.s of same period & parallel to each other (along the same path). Find the resultant amplitude when phase difference is 1) 0 2) $\pi/3$ 3) $\pi/2$ 4) π radians.

Ans : Consider a particle subjected simultaneously to two S.H.M.s having same period & travelling along the same path but of different initial phases & amplitudes.

Equations of displacements of two S.H.M.s of same period & centre.

$$x_1 = A_1 \sin(\omega t + \phi_1)$$
$$x_2 = A_2 \sin(\omega t + \phi_2)$$

where A_1 & A_2 = amplitudes, ω = angular frequency and ϕ_1 & ϕ_2 = initial phases.

The resultant displacement (x) is given by -

$$x = x_1 + x_2$$
$$= A_1 \sin(\omega t + \phi_1) + A_2 \sin(\omega t + \phi_2)$$

$\therefore x = A_1 [\sin \omega t. \cos \phi_1 + \cos \omega t. \sin \phi_1] + A_2 [\sin \omega t. \cos \phi_2 + \cos \omega t. \sin \phi_2]$

$= A_1 \sin \omega t. \cos \phi_1 + A_1 \cos \omega t . \sin \phi_1 + A_2 \sin \omega t. \cos \phi_2 + A_2 \cos \omega t. \sin \phi_2$

$\therefore x = \sin \omega t [A_1 \cos \phi_1 + A_2 \cos \phi_2] + \cos \omega t [A_1 \sin \phi_1 + A_2 \sin \phi_2]$

Let $R \cos \delta = A_1 \cos \phi_1 + A_2 \cos \phi_2$ ....(1)

& $R \sin \delta = A_1 \sin \phi_1 + A_2 \sin \phi_2$ ....(2)

$\therefore x = \sin \omega t. R \cos \delta + \cos \omega t. R \sin \delta$

$= R [\sin \omega t. \cos \delta + \cos \omega t. \sin \delta]$

$$\therefore \quad \boxed{x = R.\sin(\omega t + \delta)}$$

This is equation of S.H.M. of same angular frequency & period with amplitude R & initial phase δ. Thus resultant motion is also S.H.M.

Amplitude (R) of resultant motion :

Squaring & adding equations (1) & (2) $\Rightarrow$

$\therefore R^2 \sin^2 \delta + R^2 \cos^2 \delta$

$= (A_1 \sin \phi_1 + A_2 \sin \phi_2)^2 + (A_1 \cos \phi_1 + A_2 \cos \phi_2)^2$

$\therefore R^2 (\sin^2 \delta + \cos^2 \delta) =$
$A_1^2 \sin^2 \phi_1 + 2A_1A_2 \sin\phi_1 \sin\phi_2 + A_2^2 \sin^2\phi_2$
$+ A_1^2 \cos^2\phi_1 + 2A_1A_2 \cos \phi_1 \cos \phi_2 + A_2^2 \cos^2\phi_2$

$\therefore R^2 = A_1^2(\sin^2 \phi_1 + \cos^2 \phi_1) + A_2^2(\sin^2 \phi_2 + \cos^2 \phi_2) + 2A_1 A_2 (\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2)$

$\therefore R^2 = A_1^2 + A_2^2 + 2A_1A_2.\cos(\phi_1 - \phi_2)$

$\therefore$ Resultant amplitude -

$$\boxed{R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2.\cos(\phi_1 - \phi_2)}}$$

Initial phase (δ) :

equation(2) ÷ equation (1) =>

$$\frac{R \sin \delta}{R \cos \delta} = \frac{A_1 \sin \phi_1 + A_2 \sin \phi_2}{A_1 \cos \phi_1 + A_2 \cos \phi_2}$$

$$\therefore \tan \delta = \frac{A_1 \sin \phi_1 + A_2 \sin \phi_2}{A_1 \cos \phi_1 + A_2 \cos \phi_2}$$

$$\boxed{\delta = \tan^{-1}\left(\frac{A_1 \sin \phi_1 + A_2 \sin \phi_2}{A_1 \cos \phi_1 + A_2 \cos \phi_2}\right)} \quad \text{-------- (*)}$$

1) Phase difference = 0 :

i.e. $\phi_1 - \phi_2 = 0$ $\quad \therefore \cos(\phi_1 - \phi_2) = \cos 0 = 1$

$\therefore R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2}$

$= \sqrt{(A_1 + A_2)^2}$

$\therefore R = A_1 + A_2$

2) Phase difference = $\pi/3$ rad :

i.e. $\phi_1 - \phi_2 = \pi/3$ $\quad \therefore \cos(\phi_1 - \phi_2) = \cos \pi/3 = ½$

$\therefore R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \times \frac{1}{2}}$

$\therefore R = \sqrt{A_1^2 + A_2^2 + A_1A_2}$

3) Phase difference = $\pi/2$ rad :

i.e. $\phi_1 - \phi_2 = \frac{\pi}{2}$ $\quad \therefore \cos(\phi_1 - \phi_2) = \cos \pi/2 = 0$

$\therefore R = \sqrt{A_1^2 + A_2^2}$

4) Phase difference = π rad :

i.e. $\phi_1 - \phi_2 = \pi$ $\quad \therefore \cos(\phi_1 - \phi_2) = \cos \pi = -1$

$\therefore$ R = $\sqrt{A_1^2 + A_2^2 + 2A_1A_2\,(-1)}$

= $\sqrt{A_1^2 + A_2^2 - 2A_1A_2}$

$\therefore$ R = $|A_1 - A_2|$

Q. 15 : Two S.H.M.S are represented by x_1 =A_1 sin(ωt + $\propto_1$) & x_2 = A_2 sin (ωt + $\propto_2$). Obtain the expressions for displacement, amplitude & initial phase of resultant motion.

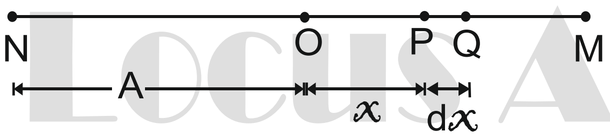
Ans : Write complete Q. No. (14) upto (*).

Q. 16 : Obtain expression for kinetic energy, potential energy & total energy of a particle performing linear S.H.M. Hence show that total energy is conserved.

OR

Show that the total energy of the particle performing linear S.H.M. is constant.

Ans : Consider a particle of mass 'm' is performing a linear S.H.M. along the path MN about the mean position O. At a given instant, let the particle is at P, at a distance x from O.



A is the amplitude of SHM. Velocity of particle is – v = ω $\sqrt{A^2 - x^2}$

Kinetic Energy is –

E_k = $\frac{1}{2}$mv^2

= $\frac{1}{2}$mω^2 $(A^2 - x^2)$

= $\frac{1}{2}$ k $(A^2 - x^2)$ …. (k = mω^2)

The restoring force acting on particle at point P is -

f = - kx { k = force constant }

Suppose the particle is displaced further by small displacement 'dx' against force f. The external work done is -

dW = f (- dx)

= (-kx)(- dx)

= kx dx

negative sign indicates that force is opposite to displacement in direction.

The total work done for displacement x is -

$W=\int_0^x dW \quad = \int_0^x kx\,.dx$

W = $\frac{1}{2}$ kx^2

This work done is stored in the form of P.E.

$\therefore$ E_p = $\frac{1}{2}$ kx^2

The total energy of particle is given by-

E = E_k + E_p

= $\frac{1}{2}$ k $(A^2 - x^2)$ + $\frac{1}{2}$ k x^2

= $\frac{1}{2}$ k $(A^2 - x^2 + x^2)$

$\therefore$ $\boxed{E = \frac{1}{2} k A^2 = \frac{1}{2} m\omega^2 A^2}$ …..(1)

As m, ω & A are constant, the total energy of the particle at any point P is constant (independent of position x & time t), the energy in S.H.M. is conserved.

Q. 17 : Show that energy of S.H.M. is directly proportional to 1) square of amplitude & 2) square of frequency.

Ans : Write upto equation(1) from Q. No. (16)

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From this equation, it is clear that 1) E ∝ ω^2 &

2) E ∝ A^2

i.e. Total energy is directly proportional to -

1) square of angular velocity &

2) square of amplitude.

Now putting ω = 2πn in equation (1) ⇒

$\therefore$ E = $\frac{1}{2}$ m x 4$\pi^2 n^2$ x A^2

$\therefore$ E = 2m$\pi^2 n^2$A^2

It is clear that

E ∝ n^2

i.e. total energy is directly proportional to square of frequency.

Q. 18 : State the expression for total energy of a particle performing SHM. Write conclusions. OR State the factors which the total energy of a particle executing SHM depends on.

Ans : The total energy of a particle performing SHM of angular frequency ω & amplitude A is –

E = $\frac{1}{2}$ k A^2= $\frac{1}{2}$ mω$^2A^2$ = $\frac{1}{2}$ m x 4$\pi^2 n^2$ x A^2

= 2π^2mn^2 A^2 = $\frac{2\,\pi^2 m\,A^2}{T^2}$

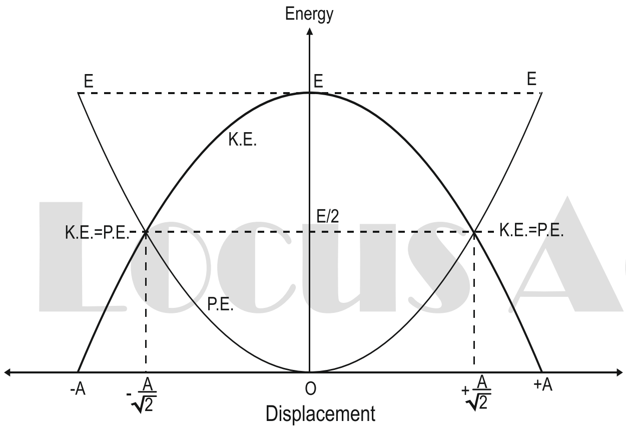
Where n = $\frac{\omega}{2\pi}$ & T = $\frac{2\pi}{\omega}$ are frequency & period of motion & k = $m\omega^2$ is force constant.

Conclusions : The total energy of particle is –

1) Independent of x & thus remains constant when m, ω & A are constant.
2) Directly proportional to force constant (k)
3) Directly proportional to mass of particle (m)
4) Directly proportional to square of amplitude (A^2)
5) Directly proportional to square of frequency (n)
6) Inversely proportional to the square of the period (T).

Q. 19 : Represent graphically the variations of K.E, P.E., & T.E. of a particle performing linear SHM with respect to it's displacement.

Ans :



Energy in SHM

Q. 20 : Define an ideal Simple pendulum. Show that under certain conditions, the simple pendulum is simple harmonic. Obtain expression for it's period. [4M – Mar'13]

OR

Show that under certain conditions, a simple pendulum performs linear S.H.M. OR Obtain expression for the period of a simple pendulum performing S.H.M.

Ans : Ideal Simple pendulum : It is a heavy particle suspended by a massless, inextensible, flexible string from a rigid support.

Consider a simple pendulum of length L, suspended from rigid support O. It is displaced through some angle θ from its mean position & released. It performs oscillations between point B & C.

The forces acting on bob in position B are -

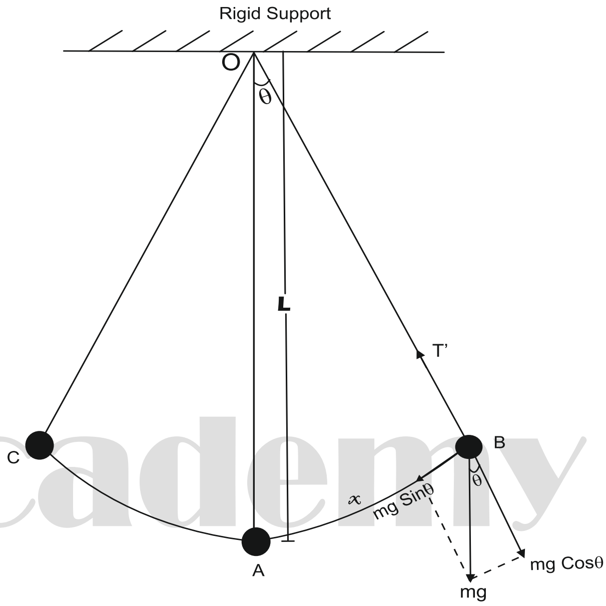
1) Force T' due to tension in string (Directed along string)

2) Weight of bob = mg (Directed vertically downward)

The weight mg is resolved as –

1) mg. cosθ parallel to length of string.

2) mg. sinθ perpendicular to length of string.



mg. cos θ is balnced by T' & mg.sin θ provides necessary restoring force to carry bob to mean position.

$\therefore$ Restoring force, F = - mg sinθ

-ve sign indicates that force is opposite to displacement in direction.

If θ is very small & in radian, Sin $\theta \cong \theta$

$\therefore$ F = - mg.θ

Now, $\theta = \frac{Arc}{radius} = \frac{x}{L}$

$\therefore$ F = - mg. $\frac{x}{L}$ = - $\frac{mg}{L}.x$ ……………(1)

As m, g & L are constant,

$$F \propto -x$$

Thus restoring force is directly proportional to the displacement. Hence motion of simple pendulum is linear S.H.M.

Now, Acceleration, a = $\frac{F}{m}$ = - $\frac{mgx/_L}{m}$ …From (1)

$= -\frac{g}{L}x$

Acceleration per unit displacement = $\left|\frac{a}{x}\right| = \frac{g}{L}$

Period of simple pendulum is -

$T = \frac{2\pi}{\omega}$

$= \frac{2\pi}{\sqrt{Acceleration\ per\ unit\ displacement}}$

$= \frac{2\pi}{\sqrt{g/L}}$

$\therefore$ $$T = 2\pi\sqrt{\frac{L}{g}}$$

Q. 21 : Show that the period of a simple pendulum is directly proportional to square root of its length & independent of weight of bob

Ans : write complete Q. No. (20)

+

As there is no term of mass of bob in above equation, the period is independent of mass of bob (m) & hence weight of bob (mg).

At a given place g is constant.

$\therefore T \propto \sqrt{L}$ [$\therefore T^2 \propto L$]

i.e. period of simple pendulum is directly proportional to square root of its length.

Q. 22 : Show that the frequency of a simple pendulum is independent of mass of bob.

Ans : Write complete Q. No. (20)

+

But frequency, $n = \frac{1}{T}$

$$\therefore \mathbf{n} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

As there is no term of mass of bob in above equation, the frequency of bob of simple pendulum is independent of mass of bob.

Note : In above equation, g is constant at given place.

$\therefore n \propto \frac{1}{\sqrt{L}}$ or $n^2 \propto \frac{1}{L}$

Q. 23 : Find the expression for periodic time of a simple pendulum. State the factors on which it depends.

Ans : Write complete Q. No. (20)

+

Write first 2 laws from Q. No. (24).

Q. 24 : State the laws of simple pendulum.

Ans : The time period of simple pendulum is given by -

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L ⇒ length of simple pendulum &

g ⇒ acceleration due to gravity at that place

The laws of simple pendulum are -

1) **Law of length :** The period of simple pendulum is directly proportional to square root of its length.

   $T \propto \sqrt{L}$

2) **Law of acceleration due to gravity :** The period of simple pendulum is inversely proportional to square root of acceleration due to gravity

   $T \propto \sqrt{\frac{1}{g}}$

3) **Law of mass :** The period of simple pendulum does not depend upon mass.

4) **Law of isochronism :** The period of simple pendulum does not depend upon on the amplitude of oscillation provided that the amplitude is small.

Q. 25 : What is seconds pendulum? Find the expression for it's length at a given place. Show that it's length has a fixed value at a given place.

Ans : Seconds pendulum : The simple pendulum of which period is two seconds is called as second's pendulum.

Period of simple pendulum, $T = 2\pi\sqrt{\frac{L}{g}}$

For second pendulum, T = 2 s

$\therefore 2 = 2\pi\sqrt{\frac{L}{g}}$

$\therefore 1 = \pi\sqrt{\frac{L}{g}}$

$\therefore 1 = \frac{\pi^2 . L}{g}$

$\therefore$ Length of second pendulum is

$$\mathbf{L} = \frac{g}{\pi^2}$$

Q. 26 : Show that length of seconds pendulum is directly proportional to acceleration due to gravity.

Ans : Write complete Q. No. (25)

+

As π^2 is constant.

$\therefore$ **L $\propto$ g**

i.e. length of seconds pendulum is directly proportional to acceleration due to gravity.

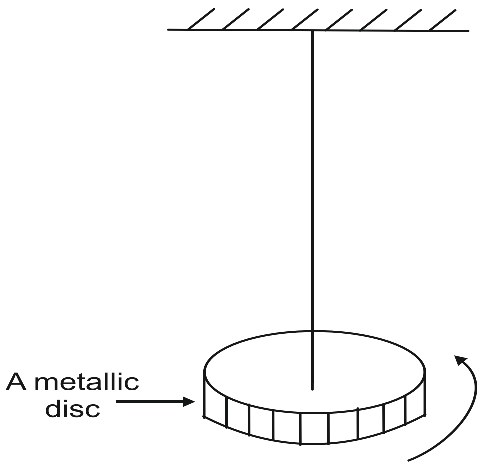
Q. 27 : Distinguish between simple pendulum & conical pendulum.

Ans :

Simple Pendulum	Conical Pendulum
1. The oscillations of the bob are in a vertical plane.	2. The bob performs UCM in a horizontal plane & string describes a cone.
2. The period is given by – $T = 2\pi\sqrt{\frac{L}{g}}$	3. The period is given by - $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$
3. The energy of bob transfers between K.E. & P.E. Total mechanical energy remains constant.	3. Gravitational P.E. of bob is constant nearly zero. Total mechanical energy is constant & it is only K.E.

Q. 28 : Define angular or torsional oscillations. Hence obtain the differential equation of the motion. Also define angular S.H.M. & find it's period.

Ans :



Suppose a disc is suspended from it's centre by a wire such that disc remains horizontal. If the disc is slightly twisted about the axis along the wire & released, it performs rotational motion partly in clockwise & anticlockwise sense. Such oscillations are called <u>angular</u> or <u>torsional</u> oscillations.

The restoring torque is always opposite to angular displacement. If it's magnitude is proportional to corresponding angular displacement, the motion is called angular SHM.

Thus for angular SHM, Restoring torque is –

$$\tau \propto -\theta$$

$$\therefore \ \tau = -c\,\theta \qquad \text{............(1)}$$

where c $\Rightarrow$ constant of proportionality.

If I is the M.I. of body, then torque acting on the body is –

$$\tau = I\alpha \qquad \text{............(2)}$$

where $\alpha \Rightarrow$ angular acceleration

From equation (1) & (2) $\Rightarrow$

$$I\alpha = -c\,\theta$$

$$I\frac{d^2\theta}{dt^2} + c\,\theta = 0 \qquad \left(\because \alpha = \frac{d^2\theta}{dt^2}\right)$$

This is differential equation of angular SHM.

$$\alpha = \frac{d^2\theta}{dt^2} = -\frac{c\theta}{I}$$

Acceleration per unit angular displ. = $\left|\frac{\alpha}{\theta}\right| = \frac{c}{I}$

Since c & I are constants, the angular acceleration is directly proportional to θ & it's direction is opposite to angular displacement. Hence, this oscillatory motion is called angular S.H.M.

Angular S.H.M. : It is defined as the oscillatory motion of a body in which torque is directly proportional to the angular displacement & it's direction is opposite to angular displacement.

The period of angular S.H.M. is

$$T = \frac{2\pi}{\omega}$$

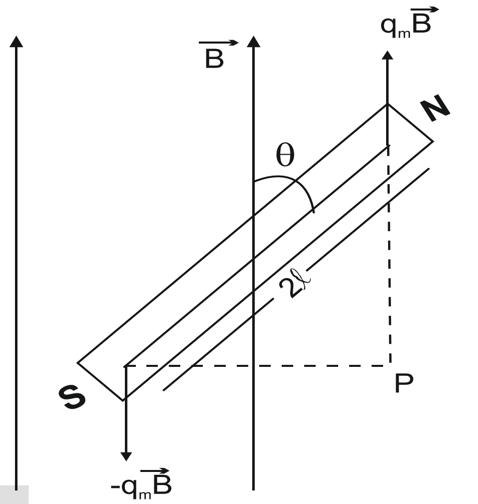
$$\therefore T = \frac{2\pi}{\sqrt{Acceleration\ per\ unit\ angular\ displacement}}$$

$$= \frac{2\pi}{\sqrt{C/I}}$$

$$\therefore \quad \boxed{T = 2\pi\sqrt{\frac{I}{c}}}$$

Q. 29 : Prove that under certain conditions a magnet vibrating in uniform magnetic field performs angular SHM.

Ans : Consider a bar magnet of magnetic moment μ, suspended horizontally by a twistless fibre. The bar magnet is free to rotate in a horizontal plane. It comes to rest in approx. the North-South direction along $\vec{B}$. If it is rotated in the horizontal plane by a small displacement θ from rest, the suspension fibre is twisted. When the magnet is released, it oscillates about the rest position in angular oscillation.



The bar magnet experiences a torque τ due to field $\vec{B}$, which tends to restore it to original position parallel to $\vec{B}$.

The restoring torque is –

$$\tau = -\mu B \sin\theta$$

If θ is small, $\sin\theta \approx \theta$

$$\tau = -\mu B\,\theta \qquad \text{........(1)}$$

$$\therefore \boxed{\tau \propto -\theta}$$

Thus, torque (hence angular acceleration) is directly proportional to angular displacement & it's direction is opposite to angular displacement.

Hence, for small angular displacement, the oscillations of the bar magnet in uniform magnetic field is simple harmonic.

Q. 30 : Obtain the expression for the period of a magnet in angular simple harmonic oscillations in a uniform magnetic field.

Ans : Write complete Q. No. (29) upto equation (1)

+

Also, $\tau = I\alpha = I\frac{d^2\theta}{dt^2}$ ....(2)

where I ⇒ M. I. of magnet

α ⇒ Angular acceleration

Form equation (1) & (2) ⇒

$$I\frac{d^2\theta}{dt^2} = -\mu B\theta$$

$$\therefore \quad I\frac{d^2\theta}{dt^2} + \mu B\theta = 0$$

$$\therefore \quad \frac{d^2\theta}{dt^2} + \frac{\mu B}{I}\theta = 0 \qquad \text{.......(3)}$$

This is equation of motion of bar magnet.

Equation (3) ⇒

$$\alpha + \frac{\mu B}{I}\theta = 0$$

$$\therefore \frac{\alpha}{\theta} = -\frac{\mu B}{I}$$

∴ Angular acceleration per unit angular displacement = $\left|\frac{\alpha}{\theta}\right| = \frac{\mu B}{I}$

∴ Period of oscillations is –

$$T = \frac{2\pi}{\sqrt{\text{Angular acceleration per unit angular displace.}}}$$

$$= \frac{2\pi}{\sqrt{\alpha/\theta}} = 2\pi\sqrt{\frac{\theta}{\alpha}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{I}{\mu B}}}$$

Q. 31 : What is meant by damped oscillations? Draw a neat labelled diagram of damped spring-and-block oscillator.

Ans : Oscillations of gradually decreasing amplitude are called damped oscillations. Oscillations of a system in the presence of dissipative frictional forces are damped. The oscillator is called a damped oscillator.

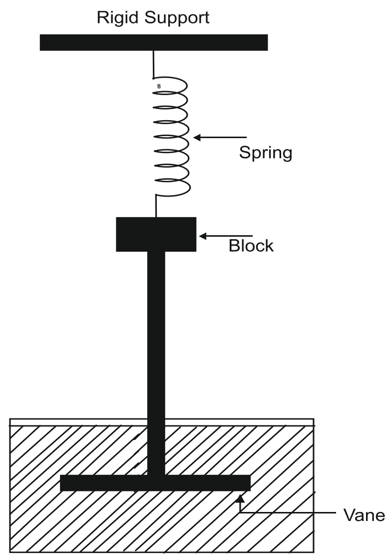


Fig. : A damped Oscillator

Figure shows a spring-and-block oscillator attached with a light Vane that moves in a fluid with viscosity. When the system is set into oscillation, the amplitude decreases for each oscillation due to the viscous drag in the vane. The mechanical energy of the block-spring system decreases with time, as energy is transferred to thermal energy of the liquid & vane.

Q. 32 : Distinguish between free vibrations & forced vibrations.

Ans :

Free vibrations	Forced vibrations
1. Free vibrations are produced when a body is disturbed from it's equilibrium position & released. eg. Simple pendulum	1. Forced vibrations are produced by an external periodic force. eg. Musical instrument having a sounding board.
2. The frequency of free vibrations depend on the body & is called natural frequency.	2. The frequency of forced vibrations is equal to that of the external periodic force.
3. The energy of the body remains constant only in the absence of friction, air resistance etc.	3. The energy of the body is maintained constant by the external periodic force.

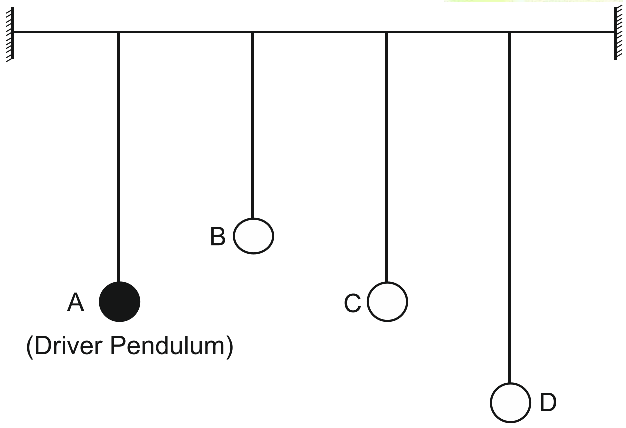
Q. 33 : Explain 1) free vibrations & 2) forced vibrations.

Ans : Write points (1) to (3) from Q. No. (32).

Q. 34 : Explain Resonance.

Ans : Resonance : If a body is made to vibrate by an external periodic force, whose frequency is equal to the natural frequency of the body, the body vibrates with maximum amplitude. This phenomenon is called resonance.

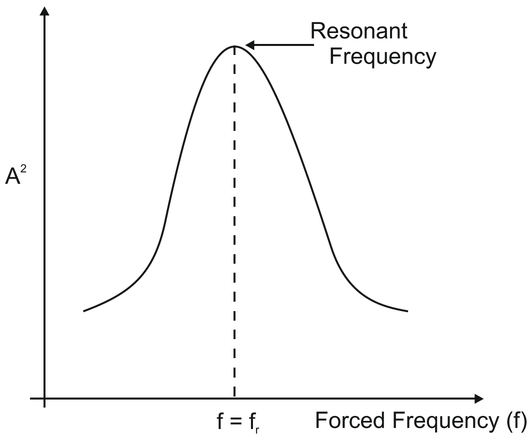
The corresponding frequency is called the resonant frequency.



Consider 4 pendulums are tied to a string. A & C are of same length, B is shorter & D is longer. Pendulum A having a solid rubber ball as bob acts as a driver or source pendulum.

As A & C are of same length, their natural frequencies are same. A is now set into oscillations perpendicular to string. In the course of time other 3 will also start oscillating.

It is observed that pendulum C oscillates with maximum amplitude & other 2 with smaller amplitude. Thus C has absorbed maximum energy. Thus it is in resonance with A.



If graph of squares of amplitudes of different pendula is plotted against their natural frequencies then peak occurs when forced frequency matches with natural frequency ie. at resonant frequency.

FORMULAE

1) Restoring force :

$F = -kx = -m\omega^2 x$ ....($\omega^2 = \frac{k}{m}$)

2) Differential equation of linear S.H.M.

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

3) Displacement in S.H.M. :

$x = A.\sin(\omega t + \Phi)$

i) Mean position, ($\Phi = 0$) $\Rightarrow x = A.\sin \omega t$

ii) Extreme position, ($\Phi = \frac{\pi}{2}$) $\Rightarrow x = A.\cos \omega t$

4) Velocity in S.H.M. :

$v = \omega \sqrt{A^2 - x^2}$

i) Mean position, ($x = 0$) $\Rightarrow v_{max} = A\omega$

ii) Extreme position, ($x = A$) $\Rightarrow v_{min} = 0$

5) Acceleration in S.H.M. :

$a = \omega^2 x$ ...(In magnitude)

i) Mean position, ($x = 0$) $\Rightarrow a_{min} = 0$

ii) Extreme position, ($x = A$) $\Rightarrow a_{max} = \omega^2 A$

6) Period of S.H.M. :

$T = 2\pi\sqrt{\frac{m}{k}}$

7) Composition of S.H.M.

i) Resultant displacement :

$x = R\sin(\omega t + \delta)$

ii) Resultant amplitude :

$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2.\cos(\Phi_1 - \Phi_2)}$

iii) Phase :

$\delta = \tan^{-1}\left(\frac{A_1\sin\Phi_1 + A_2\sin\Phi_2}{A_1\cos\Phi_1 + A_2\cos\Phi_2}\right)$

8) Energy in S.H.M. :

1) K.E. = $\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}m\,\omega^2(A^2 - x^2)$

2) P.E. = $\frac{1}{2}kx^2 = \frac{1}{2}m\,\omega^2 x^2$

3) T.E. = K.E. + P.E. = $\frac{1}{2}k\,A^2 = \frac{1}{2}m\,\omega^2 A^2$

9) Period of simple pendulum :

$T = 2\pi\sqrt{\frac{L}{g}}$

10) Length of seconds pendulum :

$L = \frac{g}{\pi^2}$

11) Period of torsional oscillation (angular S.H.M.)

$T = 2\pi\sqrt{\frac{I}{C}}$

12) Period of magnet performing angular acceleration :

$T = 2\pi\sqrt{\frac{I}{\mu B}}$

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